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ABSTRACT

Modern astrogeodetic methods, although accurate, are inefficient and too complex to rapidly determine deflections of the vertical. This problem is the impetus for finding a more useful technique that can yield results much more quickly. The Global Positioning System (GPS), with its ability to provide coordinate differences in interferometric modes, can be used to determine these deflections. Using highly accurate coordinate differences in conjunction with orthometric height differences, one can develop a surface of the geoidal undulations as a function of latitude and longitude. Given three GPS stations, a local surface can only be approximated by a plane. With more points, however, the modeled surface will more accurately resemble the true undulation differences. From this modeled surface, one uses least squares fitting of polynomials to interpolate the "average" and in the survey area. Finally, the deflections and are computed and a study of propagation of both absolute and relative errors is made.

DETERMINATION OF DEFLECTIONS OF THE VERTICAL USING THE GLOBAL POSITIONING SYSTEM

A Technical Report

Presented in partial fulfillment of the requirements for
the Masters of Science degree in the
Department of Geodetic Science and Surveying
at the Ohio State University

by

Andrey Aristov

* * * * * *

The Ohio State University
November 1987

Approved by

Advisor

Dept. of Geodetic Science
and Surveying

ABSTRACT

Modern astrogeodetic methods, although accurate, are inefficient and too complex to rapidly determine deflections of the vertical. This problem is the impetus for finding a more useful technique that can yield results much more quickly. The Global Positioning System (GPS), with its ability to provide coordinate differences in interferometric modes, can be used to determine these deflections. Using highly accurate coordinate differences in conjunction with orthometric height differences, one can develop a surface of the geoidal undulations as a function of latitude and longitude. Given three GPS stations, a local surface can only be approximated by a plane. With more points, however, the modeled surface will more accurately resemble the true undulation differences. From this modeled surface, one uses least squares fitting of polynomials to interpolate the "average" $\frac{\partial N}{\partial \phi}$ and $\frac{\partial N}{\partial \lambda}$ in the survey area. Finally, the deflections ξ and η are computed and a study of propagation of both absolute and relative errors is made.

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Dr. Richard H. Rapp gave me data against which I could compare my results and also periodically exposed me to new points of view which I would not have considered otherwise. Finally, I need to thank some of my fellow students who helped me with the field work (although true observations never really materialized): David Chadwell, Harley Pitt, and Paul Harwig.

INTRODUCTION

Current astrogeodetic techniques employ well-known geodetic concepts with extremely accurate, complex and expensive astronomical equipment. Combining geodetic coordinates from conventional surveys such as triangulation, trilateration or traverse with astronomically observed natural coordinates, one can determine deflections of the vertical for many points on the solid surface of the earth. The relationship between geodetic and natural coordinates is:

$$\begin{aligned} \phi &= \Phi - \xi \\ \lambda &= \Lambda - \eta \sec \phi \end{aligned} \tag{1}$$

(Heiskanen and Moritz, 1967) where ξ is the deflection in the north-south direction and η is the deflection in the east-west direction.

Obtaining accurate geodetic coordinates with conventional methods is not an easy task. A skilled crew of four to five surveyors must spend a significant amount of time "in the field" connecting previously-established control to the point or points of interest. Depending on the distance between the existing control and the new stations and on the roughness and vegetation of the terrain, the crew may need many days or even weeks to

obtain the required data. Furthermore, observing stars to determine natural coordinates with astrogeodetic equipment can be accurately completed by only the most accomplished of astronomic surveyors. Because the equipment (usually a Wild T-4 in the United States) is so heavy and sensitive, many areas do not qualify for astronomic observations. Rough mountainous regions, swampland and glaciated areas cannot adequately provide support or shelter for the telescopes. Urban areas, polluted skylines or hazy atmospheres are also generally out of the astronomer's range because of the inadequacy of the light coming in from the stars. All in all, there is no question that modern astrogeodetic techniques are burdened by a plethora of difficulties.

Observations with the Global Positioning System (GPS), on the other hand, are tremendously advantageous in that they defeat many of the problems that plague astronomic techniques. Firstly, GPS receivers are continuously being ruggedized and can already withstand conditions many times more severe than what an astrotheodolite can undergo. Secondly, GPS observations are not dependent upon weather conditions -- clouds, high winds, extremes of temperature, rain and even snow do not significantly hamper field collection of data. Most importantly, however, is the rapidity with which GPS data can be obtained. Whereas astrogeodetic collection may take several days (or weeks if the weather is uncooperative), a GPS operator gathers all of his data, which includes a sufficient amount of redundancy, in only a few hours. Furthermore, while it may take months to properly train an

astrogeodesist on a T-4, a GPS operator can be taught his procedures in a single afternoon. No special skills are required to collect GPS data; a good astrogeodesist, however, is a rare specialist.

A very noteworthy aspect of GPS data collection is that most of the work is performed "in the lab" and not in the field. With high-speed computers and highly efficient software, the more expensive and cumbersome conventional field techniques may eventually become obsolete. As GPS receivers become more affordable, many optical surveying methods will slowly die out. In this paper, I shall show how collection of coordinate difference data with GPS receivers can be used to determine deflections of the vertical accurately, efficiently and inexpensively.

SURVEY GEOMETRY

Although there is an infinite variety of survey layouts and geometries, networks used for accurate determinations of deflections of the vertical will impose some, although rather flexible, rules on the distribution of the survey points. One of the few practical applications for deflection computations is found in the determination of launch and initial trajectory characteristics of missiles and rockets. The inertial guidance systems of these space or sub-space vehicles must be aligned with the local vertical in order to adequately predict their paths and orbital characteristics. Even a small error in the computation of the plumb line may result in a significant deviation of the predicted orbital path from the actual one. (Mueller, 1969; Kaula, 1966)

An optimum survey will include one station right over or very near the point at which the deflection of the vertical is sought. From this point would extend radials of about one to fifteen kilometers in length to other peripheral stations. The appearance of such a plan would resemble the spokes of a wheel. (See Figure 1, next page.) It is not important that these "spokes" be of equal length or be equally spaced although an even distribution is always favorable.

In highly benign or smooth gravity fields, only two peripheral stations may be sufficient. As the complexity of the

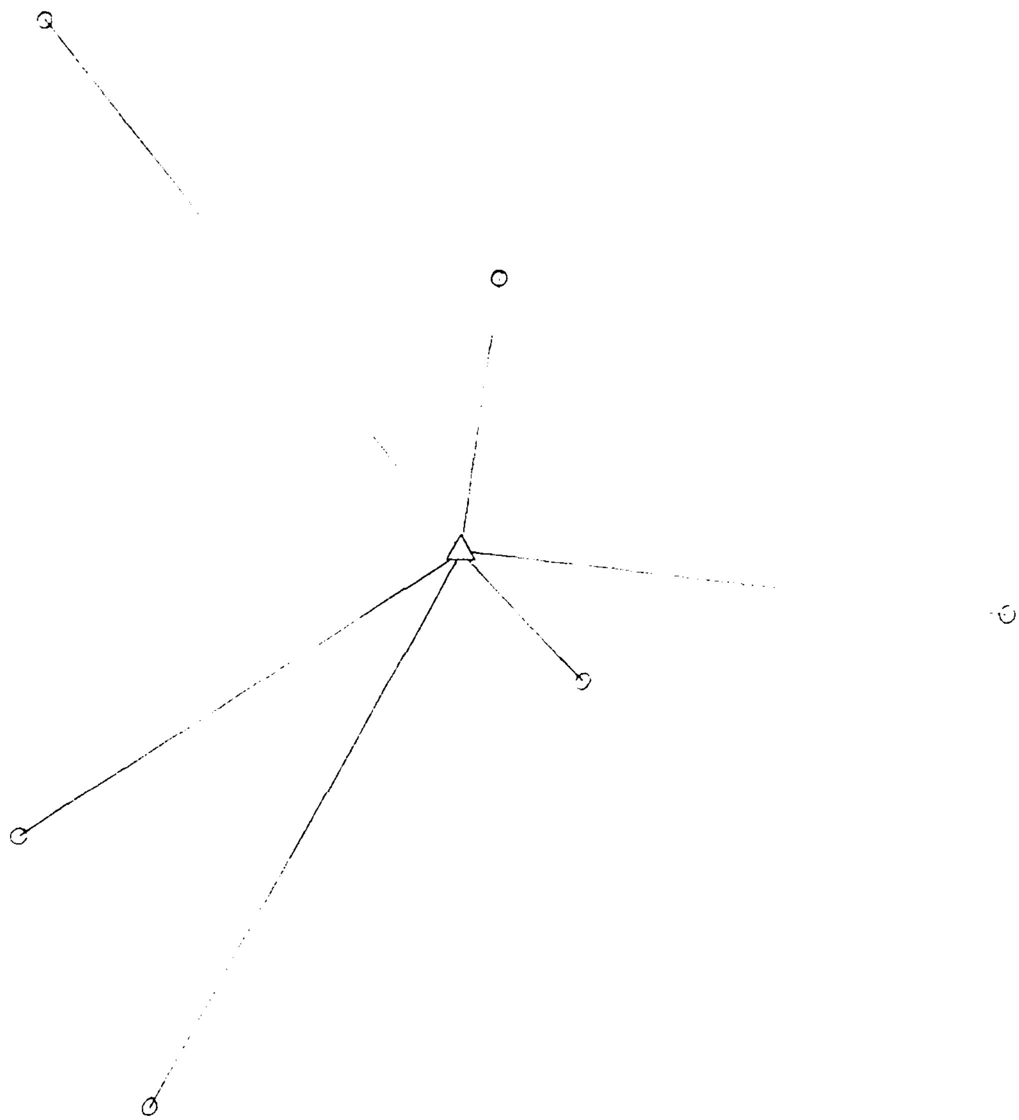


Figure 1.

Plan view of a possible survey layout for determination of deflections of the vertical using GPS. Triangle indicates the central point for which the deflection is sought. Circles are the "peripheral" stations to which GPS measurements are made. Both GPS observations and high-order leveling are required at all of the survey points.

field increases, more points will become necessary to adequately model and interpolate the data. If the field is extremely complex, it is possible that so many stations will be required that this GPS technique will become less efficient than the existing astrogeodetic methods. In most cases, however, GPS surveying will be much more advantageous than the astronomical methods used today.

LEAST SQUARES ADJUSTMENTS

If more than two GPS receivers are used over several separate sessions to cover the survey area, then the least squares adjustments will have to employ more complex techniques than the ones described in this paper. (Double and triple differencing will definitely come into use, for instance.) (Goad, 1987; Bock et al, 1986; Beutler et al, 1986) In this paper, I will assume for simplicity's sake that, in fact, a radial survey is conducted with all observations on all survey marks made simultaneously. Expansion to the more complex situations is a subject already well-documented by many other authors. (See references)

Using the notation of Uotila (1986) and the algorithm of Sliwa (1985), the mathematical model that we employ is one of observation equations:

$$L^a = F(X^a) = F(X^o) + \left.\frac{\partial F}{\partial X^a}\right|_{X^a=X^o} X + \left.\frac{\partial F}{\partial X^a}\right|_{X^a=X^1} X + \cdots \qquad (2)$$

where L^a = adjusted values of the observed quantities

X^a = theoretical values of the unknown parameters.

In the relative mode of GPS surveying, one observes coordinate differences between stations: ΔX_{ij}, ΔY_{ij} and ΔZ_{ij} and eventually solves for the absolute (though not necessarily accurate) positions of those stations: $X_{i...j}$, $Y_{i...j}$, $Z_{i...j}$ where i and j are the station identifiers. (Sliwa, 1985) Thus, expanding the mathematical model from the generalized form above yields:

$$\begin{aligned}\Delta X_{ija} &= X_{ja} - X_{ia} \\ \Delta Y_{ija} &= Y_{ja} - Y_{ia} \\ \Delta Z_{ija} &= Z_{ja} - Z_{ia}\end{aligned} \tag{3}$$

where station "i" is the central point and all stations "j" are the peripheral marks.

One obtains initial approximations for the parameters X^o by removing all known errors (blunders, atmospheric effects, satellite and receiver clock errors, receiver measurement noise, etc.) (Wei, 1985) The coefficient matrix derived from model linearization (Sliwa, 1985) is then computed by differentiating the model equations with respect to the parameters:

$$A_o = \left.\frac{\partial F}{\partial X^a}\right|_{X^a = X^o} \tag{4}$$

A vector of approximate values of the observables L_o is rigorously computed by:

$$L_o = F(X^o) - L^b \tag{5}$$

where L^b is a vector of the observed values of the observables. Since the normal equations can be represented by:

$$A_o^T P A_o \hat{X}_o + A_o^T P L_o = 0 \tag{6}$$

we get the solutions for the unknowns to be:

$$\hat{X}_o = -(A_o^T P A_o)^{-1} A_o^T P L_o \tag{7}$$

where P is a weight matrix. (Uotila, 1986) In multi-receiver simultaneous surveying, there is a very high level of correlation between observations which have common points. Therefore, off-diagonal values in the weight matrix will not be equal to zero but in fact may be very significant factors in the computations.

Proper magnitudes for these correlation coefficients are well documented in the literature. (Sliwa, 1985)

A set of adjusted values $\hat{X}_0^a$ is obtained by adding the computed solutions for the unknowns $\hat{X}_0$ to the initial approximations X^o :

$$\hat{X}_0^a = X^o + \hat{X}_0 \tag{8}$$

Then we compute directly:

$$V_0^T P V_0 = L_0^T P L_0 + \hat{X}_0^T A_0^T P L_0 \tag{9}$$

and

$$\hat{\sigma}_0^2 = \frac{V_0^T P V_0}{n-u} \tag{10}$$

where $\hat{\sigma}_0^2$ is the a posteriori variance of unit weight

n is the number of observations

u is the number of parameters

$n-u$ is the degree of freedom (Uotila, 1986)

One finally computes the residual vector V_0 and the adjusted values of the observables $\hat{L}_0^a$ by:

$$V_0 = A_0 \hat{X}_0 + L_0 \tag{11}$$

$$\hat{L}_0^a = F(\hat{X}_0^a) \tag{12}$$

Iteration continues by computing new A and L matrices and determining better adjusted values, thusly:

$$A_1 = \left.\frac{\partial F}{\partial X^a}\right|_{X^a = X^1} ; \qquad X^1 = \hat{X}_0^a$$

$$L^1 = F(X^1) \tag{13}$$

$$L_1 = L^1 - L^b \qquad \text{and so forth}$$

The iterative process of this adjustment will continue until

1) $\hat{X}_i$ approaches zero

2) $(V_i - V_{i-1})$ approaches zero (14)

3) $V_i^T P V_i$ stabilizes

4) $\hat{L}_i^a = L_b + V_i = F(\hat{X}_i^a)$ (Uotila, 1986)

The adjusted values of the observables $\hat{L}_i^a$ may then be used in conjuction with a nominal set of X, Y, Z values from any one (preferably the central) station in the survey network to obtain the X, Y, Z positions of all the other stations in that network. It is very important to realize that these absolute coordinates for the stations contain errors far too great for our computations. All of the stations, however, have essentially the same absolute errors; therefore, their relative errors are very small. We shall eventually use only the highly accurate relative coordinates (coordinate differences) for determination of the deflections.

COMPUTATION OF THE DEFLECTION OF THE VERTICAL

We must first convert the absolute cartesian coordinates (X,Y,Z) into geodetic coordinates (ϕ, λ, h) using well known transformations presented by Heiskanen and Moritz (1979), Rapp (1984) and Goad (personal communication, 1986,1987):

$$\begin{aligned} X &= (N + h) \cos\phi \cos\lambda \\ Y &= (N + h) \cos\phi \sin\lambda \\ Z &= (N(1-e^2) + h) \sin\phi \end{aligned} \qquad (15)$$

where

$$N = \frac{a}{(1 - e^2 \sin^2\phi)^{1/2}}$$ = the length of the normal vector from the surface of the ellipsoid to the minor axis. (Rapp, 1984)

Geodetic longitude can be directly computed by division of the second equation by the first (keeping in mind the quadrant of the computation to maintain true signs):

$$\lambda = \mathrm{DATAN2}(Y,X) \qquad (16)$$

Computation of the geodetic latitude ϕ and height above the ellipsoid h, on the other hand, requires a Newton-Rhapson iteration. From Goad (personal communication, 1987), we have:

$$\begin{aligned} p(\phi,h) &= p(\phi_o,h_o) + p_\phi\Delta\phi + p_\lambda\Delta\lambda + \dots \\ &\cong p(\phi_o,h_o) - (N + h)\sin\phi\Delta\phi + \cos\phi\Delta h \end{aligned} \qquad (17)$$

also

$$\begin{aligned} \Delta\phi &= \frac{\cos\phi\Delta Z - \sin\phi\Delta p}{N + h} \\ \Delta h &= \sin\phi\Delta Z + \cos\phi\Delta p \end{aligned} \qquad (18)$$

where

$$p = (N + h)\cos\phi$$
$$Z = (N(1 - e^2) + h)\sin\phi \tag{19}$$

where $\Delta\phi$ and Δh are used to update the best known ϕ and h quantities. Initial values for the latitude and height (ϕ_o and h_o) are also derived by Goad (personal communication, 1987):

$$\phi_o = \sin^{-1}\frac{Z}{(p^2 + Z^2)^{1/2}}$$
$$h_o = (p^2 + Z^2)^{1/2} - a(1 - f\sin^2\phi_o) \tag{20}$$

where f is the flattening and a is the semi-major axis of the reference ellipsoid.

Once we have the absolute geodetic coordinates of all of the survey points, we compute the highly accurate coordinate differences in this same geodetic reference frame. (i.e. $\Delta\phi$, $\Delta\lambda$, Δh from the central station to every peripheral station in the network.) As mentioned before, although the absolute coordinates of the stations themselves are not acceptable, ($\pm$ 20 meters or even greater), this is of no concern since we shall only employ the coordinate differences in our algorithm. Even though the central station may have a large absolute positioning error of ϕ_e, λ_e, h_e , all of the other stations measured relative to this central one will have essentially the same ϕ_e, λ_e, h_e errors. (See Figure 2, next page.)

$(\phi + \phi_e,\ \lambda + \lambda_e,\ h + h_e)$ $(\phi+\Delta\phi+\phi_e, \lambda+\Delta\lambda+\lambda_e, h+\Delta h+h_e)$

$\Delta\phi,\ \Delta\lambda,\ \Delta h$

central station i △————————————————△ peripheral station j

Figure 2.

This absolute error will likewise translate into the N's but not the dN's of the survey. I shall show later that both the small errors in relative positioning and the large errors in absolute positioning of the points are almost always negligible in the computation of the deflections.

The geoidal undulation for each point can be computed using the difference between the geometric height above the ellipsoid (h) and the orthometric height above the geoid (H):

$$N_i = h_i - H_i \tag{21}$$

where the orthometric heights can be obtained by conventional high-order leveling techniques.

Because we only have highly accurate coordinate differences between the stations, we must not use the absolute geoidal undulations but rather the more accurate relative undulation differences:

$$\begin{aligned} dN_{ij} &= (h_j - H_j) - (h_i - H_i) \\ &= (h_j - h_i) - (H_j - H_i) \\ &= dh_{ij} - dH_{ij} \end{aligned} \tag{22}$$

$$dN_{ij} = N_j - N_i \tag{23}$$

At this point, we have very good $\Delta\phi$, $\Delta\lambda$, Δh and dN_{ij}'s for all of the lines in the network. Given only this data, we can model the surface of the geoidal differences using interpolation by least squares fitting of polynomial surfaces. The specific algorithms that we employ will be dependent upon the number of survey points in our network. According to Lancaster (1986), the general form of a quadratic function in two variables is:

$$p(x,y) = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 \qquad (24)$$

If f_i is the geoidal undulation difference between stations, then we want to adjust the function p by choosing the constants a_1 to a_6 so as to minimize

$$E(p) = \sum_{i=0}^{n} (p(x_i, y_i) - f_i)^2 \qquad (25)$$

(Lancaster, 1986)

This function E will have a minimum only when $\frac{\partial E}{\partial a_i} = 0$ for i = 1 to 6 and will yield six linear equations as functions of the a's. The six normal equations are:

$$\begin{aligned}
&(N+1)a_1 + (\Sigma x_i)a_2 + (\Sigma y_i)a_3 + (\Sigma x_i^2)a_4 \qquad (26)\\
&\quad + (\Sigma x_i y_i)a_5 + (\Sigma y_i^2)a_6 = \Sigma f_i,\\
&(\Sigma x_i)a_1 + (\Sigma x_i^2)a_2 + (\Sigma x_i y_i)a_3 + (\Sigma x_i^3)a_4\\
&\quad + (\Sigma x_i^2 y_i)a_5 + (\Sigma x_i y_i^2)a_6 = \Sigma x_i f_i,\\
&(\Sigma y_i)a_1 + (\Sigma x_i y_i)a_2 + (\Sigma y_i^2)a_3 + (\Sigma x_i^2 y_i)a_4\\
&\quad + (\Sigma x_i y_i^2)a_5 + (\Sigma y_i^3)a_6 = \Sigma y_i f_i,\\
&(\Sigma x_i^2)a_1 + (\Sigma x_i^3)a_2 + (\Sigma x_i^2 y_i)a_3 + (\Sigma x_i^4)a_4\\
&\quad + (\Sigma x_i^3 y_i)a_5 + (\Sigma x_i^2 y_i^2)a_6 = \Sigma x_i^2 f_i,\\
&(\Sigma x_i y_i)a_1 + (\Sigma x_i^2 y_i)a_2 + (\Sigma x_i y_i^2)a_3 + (\Sigma x_i^3 y_i)a_4\\
&\quad + (\Sigma x_i^2 y_i^2)a_5 + (\Sigma x_i y_i^3)a_6 = \Sigma x_i y_i f_i,\\
&(\Sigma y_i^2)a_1 + (\Sigma x_i y_i^2)a_2 + (\Sigma y_i^3)a_3 + (\Sigma x_i^2 y_i^2)a_4\\
&\quad + (\Sigma x_i y_i^3)a_5 + (\Sigma y_i^4)a_6 = \Sigma y_i^2 f_i,
\end{aligned}$$

where the summations are from 1 to n, the number of stations.

The surface that we need to model is, of course, the set of vectors defining the geoidal undulation differences (dN's) as functions of the coordinate differences ($\Delta\phi$'s and $\Delta\lambda$'s). Therefore, our p(x,y) function takes on the form:

$$p(\Delta\phi,\Delta\lambda) = \Delta N(\phi_o,\lambda_o) + \frac{\partial N}{\partial \phi}\Delta\phi + \frac{\partial N}{\partial \lambda}\Delta\lambda + \frac{\partial^2 N}{\partial \phi^2}\Delta\phi^2 + \frac{\partial^2 N}{\partial\phi\partial\lambda}\Delta\phi\Delta\lambda + \frac{\partial^2 N}{\partial\lambda^2}\Delta\lambda^2 \quad (27)$$

or

$$\begin{aligned} a_1 &= \Delta N(\phi_o,\lambda_o) \\ a_2 &= \frac{\partial N}{\partial \phi} \\ a_3 &= \frac{\partial N}{\partial \lambda} \\ a_4 &= \frac{\partial^2 N}{\partial \phi^2} \\ a_5 &= \frac{\partial^2 N}{\partial\phi\partial\lambda} \\ a_6 &= \frac{\partial^2 N}{\partial \lambda^2} \end{aligned} \qquad (28)$$

Of greatest importance, as we shall see, are the second and third terms $\frac{\partial N}{\partial \phi}$ and $\frac{\partial N}{\partial \lambda}$. The quantity $\Delta N(\phi_o,\lambda_o)$, because of the relative nature of our surface, will be an unmeasurable and undefined variable. Since we are essentially setting a zero-plane through the central point, however, this result will play no part in our final computation. In general, $\Delta N(\phi_o,\lambda_o)$, will usually be close to zero.

We can express the normal equations shown above in matrix form. From Lancaster (1986):

$$\text{let } V = \begin{bmatrix} 1 & \Delta\phi_1 & \Delta\lambda_1 & \Delta\phi_1^2 & \Delta\phi_1\Delta\lambda_1 & \Delta\lambda_1^2 \\ 1 & \Delta\phi_2 & \Delta\lambda_2 & \Delta\phi_2^2 & \Delta\lambda_2\Delta\lambda_2 & \Delta\lambda_2^2 \\ \vdots & & & & & \vdots \\ 1 & \Delta\phi_n & \Delta\lambda_n & \Delta\phi_n^2 & \Delta\phi_n\Delta\lambda_n & \Delta\lambda_n^2 \end{bmatrix} \qquad (29)$$

and the vectors:

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} \Delta N(\phi_o, \lambda_o) \\ \frac{\partial N}{\partial \phi} \\ \frac{\partial N}{\partial \lambda} \\ \frac{\partial^2 N}{\partial \phi^2} \\ \frac{\partial^2 N}{\partial \phi \partial \lambda} \\ \frac{\partial^2 N}{\partial \lambda^2} \end{bmatrix} \qquad \vec{f} = \begin{bmatrix} dN_1 \\ dN_2 \\ \vdots \\ dN_n \end{bmatrix} \tag{30}$$

The normal equations then take on the form:

$$V^T V \vec{a} = V^T \vec{f} \tag{31}$$

or, to facilitate a solution:

$$\vec{a} = (V^T V)^{-1} V^T \vec{f} \tag{32}$$

As a warning, one should be very aware of the possibility of *creating ill-conditioned* V^TV matrices caused by factorization of the coefficient matrix or by a close-to-orthogonal (pathological) survey network. (Lancaster, 1986) If the network must be of an orthogonal design, then one would advance to a more complex interpolation routine involving cubic or bicubic splines. (Lancaster, 1986)

An advantage of the above system is that one may, if one has sufficiently accurate information, impose a weight matrix (P) on the normal equations to obtain:

$$\vec{a} = (V^T P V)^{-1} V^T P \vec{f} \tag{33}$$

The diagonal coefficients in this weight matrix could be dependent upon the distances between points, the accuracy with which various parts of the field surveys are conducted, recency of the collected

data, and so forth. In this paper, however, I shall assume no such knowledge and constrain the P matrix to the identity matrix.

Interpolation with three stations:

The number of survey points observed will determine which case of the polynomial should be used. If we only observe two peripheral stations (for a total of three survey points), then we cannot use the polynomial form discussed above but must rather revert to a highly shortened version of it. With three non-collinear survey points, we can only construct a plane to approximate the undulation difference surface. (See Figure 3.) The equation of such a plane is

$$dN(\Delta\phi,\Delta\lambda) = \Delta N(\phi_0,\lambda_0) + \frac{\partial N}{\partial\phi}\,\Delta\phi + \frac{\partial N}{\partial\lambda}\,\Delta\lambda \tag{34}$$

where ϕ_0,λ_0 and $\Delta N(\phi_0,\lambda_0)$ are undefined. (Thus we can set them a priori equal to zero.) According to Lancaster (1986), we first compute Δ where

$$\Delta = \det\begin{bmatrix} 1 & \phi_0 & \lambda_0 \\ 1 & \Delta\phi_1 & \Delta\lambda_1 \\ 1 & \Delta\phi_2 & \Delta\lambda_2 \end{bmatrix} = \Delta\phi_1\Delta\lambda_2 - \Delta\phi_2\Delta\lambda_1 \tag{35}$$

then we get directly:

$$\frac{\partial N}{\partial\phi} = \Delta^{-1}\det\begin{bmatrix} 1 & N(\phi_0,\lambda_0) & \lambda_0 \\ 1 & dN_1 & \Delta\lambda_1 \\ 1 & dN_2 & \Delta\lambda_2 \end{bmatrix} = \Delta^{-1}\,[dN_1\Delta\lambda_2 - dN_2\Delta\lambda_1] \tag{36}$$

and

$$\frac{\partial N}{\partial\lambda} = \Delta^{-1}\det\begin{bmatrix} 1 & \phi_0 & N(\phi_0,\lambda_0) \\ 1 & \Delta\phi_1 & dN_1 \\ 1 & \Delta\phi_2 & dN_2 \end{bmatrix} = \Delta^{-1}\,[dN_2\phi_1 - dN_1\phi_2] \tag{37}$$

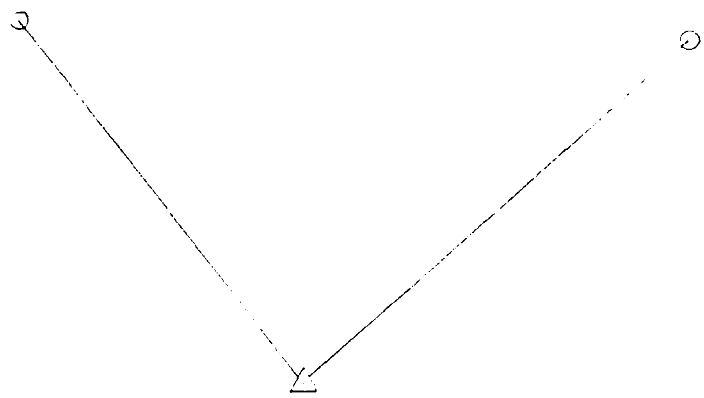


Figure 3.

Plan view of a survey network containing only two peripheral stations (circles). In this case, the geoidal undulation difference surface can only be approximated by a plane.

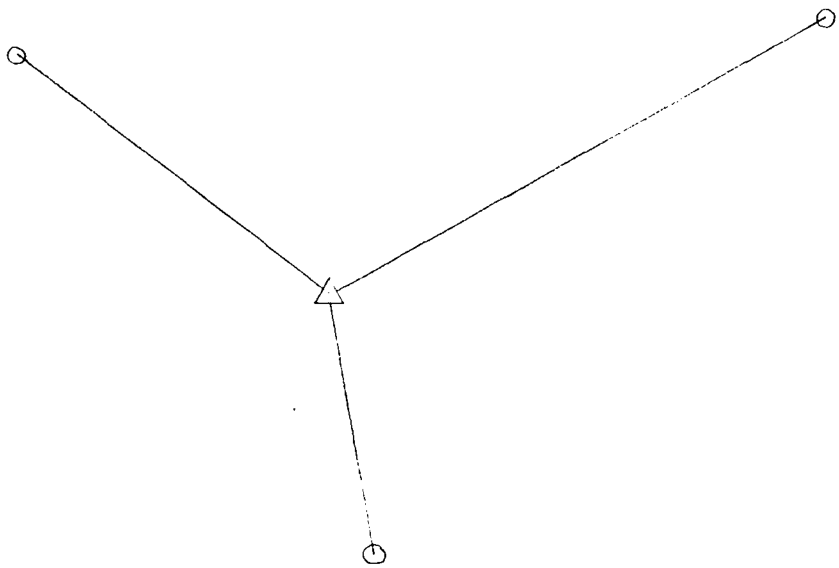


Figure 4.

Survey network containing three peripheral stations (for a total of four survey marks). The geoidal undulation difference surface will be approximated by a curved surface derived by least squares adjustments of polynomials. Similar layouts will be made for four or more peripheral stations.

Interpolation with four stations:

For a situation involving three peripheral points (four stations in all -- see Figure 4.), we cannot (usually) approximate the surface as a plane. We can, however, use a more advanced version than the one above. Once again we have:

$$dN(\Delta\phi, \Delta\lambda) = \Delta N(\phi_o, \lambda_o) + \frac{\partial N}{\partial \phi} \Delta\phi + \frac{\partial N}{\partial \lambda} \Delta\lambda \qquad (38)$$

but in this instance, $\Delta N(\phi_o, \lambda_o)$ cannot be set a priori equal to zero. The V matrix that we described before will be a 3x3 of the form

$$V = \begin{bmatrix} 1 & \Delta\phi_1 & \Delta\lambda_1 \\ 1 & \Delta\phi_2 & \Delta\lambda_2 \\ 1 & \Delta\phi_3 & \Delta\lambda_3 \end{bmatrix} \qquad (39)$$

and

$$\vec{a} = \begin{bmatrix} \Delta N(\phi_o, \lambda_o) \\ \frac{\partial N}{\partial \phi} \\ \frac{\partial N}{\partial \lambda} \end{bmatrix} \qquad \vec{f} = \begin{bmatrix} dN_1 \\ dN_2 \\ dN_3 \end{bmatrix} \qquad (40)$$

Here, we can immediately compute

$$\vec{a} = (V^T V)^{-1} V^T \vec{f} \qquad (41)$$

Once again, $\frac{\partial N}{\partial \phi}$ and $\frac{\partial N}{\partial \lambda}$ will be directly resolved.

Interpolation with five or more stations:

Finally, there is the most generalized case in which we observe four or more peripheral stations in addition to the one

central station. Now, the equation of the surface to be modeled is, as before:

$$dN(\Delta\phi,\Delta\lambda) = \Delta N(\phi_o,\lambda_o) + \frac{\partial N}{\partial\phi}\Delta\phi + \frac{\partial N}{\partial\lambda}\Delta\lambda + \frac{\partial^2 N}{\partial\phi^2}\Delta\phi^2 + \frac{\partial^2 N}{\partial\phi\partial\lambda}\Delta\phi\Delta\lambda + \frac{\partial^2 N}{\partial\lambda^2}\Delta\lambda^2 \quad (42)$$

and the matrices V, f and a are of the same form as in equations 29 and 30. The solution to this model is:

$$\vec{a} = (V^TV)^{-1}V^T\vec{f} \quad (43)$$

where $\frac{\partial N}{\partial\phi}$ and $\frac{\partial N}{\partial\lambda}$ can be directly solved. The additional terms $\frac{\partial^2 N}{\partial\phi^2}$, $\frac{\partial^2 N}{\partial\phi\partial\lambda}$ and $\frac{\partial^2 N}{\partial\lambda^2}$ will not be necessary in the final computation of the deflections, but they will "absorb" minor deviations that would otherwise be erroneously dumped into $\frac{\partial N}{\partial\phi}$ and $\frac{\partial N}{\partial\lambda}$. (This argument may, of course, be taken to extremes by computing still higher derivatives of the geoidal undulations if even more points are observed, but these corrections will truly be infinitesmal.)

Once $\frac{\partial N}{\partial\phi}$ $\frac{\partial N}{\partial\lambda}$ and an average latitude ϕ for the survey network are all computed, the determination of the deflection of the vertical components is very simple. From Heiskanen and Moritz (1979), we have in a north-south direction:

$$\xi = \varepsilon\phi = -\frac{dN}{dS_\phi} = -\frac{1}{R}\frac{\partial N}{\partial\phi} \quad (44)$$

in an east-west direction:

$$\eta = \varepsilon\lambda = -\frac{dN}{dS_\lambda} = -\frac{1}{R\cos\phi}\frac{\partial N}{\partial\lambda} \quad (45)$$

One may use algebra and trigonometry to obtain the total deflection magnitude and azimuth:

$$\varepsilon = \sqrt{\xi^2 + \eta^2} \qquad \text{(magnitude)} \quad (46)$$

$$\alpha_\varepsilon = \mathrm{DATAN2}(\xi,\eta) \qquad \text{(azimuth)} \quad (47)$$

ERROR ANALYSIS

In order to confirm the validity of the arguments made here and to check the accuracy of the deflections, a thorough error analysis must be performed on results obtained using this GPS technique. Unfortunately, a last-minute failure in one of the Geodetic Science Department's Kaypro 2000 computers (which is used in conjunction with the Trimble model GPS receiver), prevented any collection of field data. For this reason, GPS and leveling data collected and adjusted for another project must be used in our analysis. Although not completely valid, these tests should at least provide some idea as to the accuracies one might expect if one were to obtain actual GPS data for the determination of the deflection of the vertical.

The consensus in current literature lists the accuracy of GPS-determined coordinate differences to range from 1 to 3 ppm. (Goad, personal communication, 1987) Goad (1987) has used a formula of 3mm + 1×10^{-6} x(L) for computing the error in baselines of length L. The 3mm factor comes from "noise" in the system. An "optimum" length baseline, however, is still a debatable concept and should not, at least for now, be specified. As mentioned in the survey layout section of this report, baselines from 1 to 15 kilometers should be sufficient to adequately model the geoidal undulation difference surface. (Lines of 1 to 5 kilometers in length, however, are probably most appropriate for our interpolation schemes.)

A value must be placed on the desired accuracy for the deflection of the vertical. Current astrogeodetic techniques, using Wild T-4 astrotheodolites, are capable of reaching values in deflections of ± .5 seconds of arc. (Mueller, personal communication, 1987) GPS methods must therefore accomplish at least this accuracy as well. (We recall that the purpose of using GPS for determinations of deflections of the vertical is not to increase accuracy, but rather to increase efficiency and cost-effectiveness of making such computations, therefore we are not searching for better results.)

Using GPS data collected in the Columbus, Ohio area by the Franklin County Engineers Office in March 1987 and adjusted / analyzed by Dr. Goad's multi-receiver TRIMVEC program (WGS-84 parameters), we have:

	Name	Elevation
Station A (Central station)	Britton	[illegible].59 m
Station B (Peripheral station)	18-83	2[illegible]7.74 m
Station C (Peripheral station)	Clark	273.50 m

For line A-B (Britton to 18-83), we compute the weighted mean (from two observation sessions):

$$
\begin{aligned}
d\phi &= -0.089487058 \text{ deg} \\
d\lambda &= 0.094818595 \text{ deg} \\
dh &= -54.0591445 \text{ m} \\
dH &= -53.85 \text{ m} \\
dN &= -0.2091 \text{ m}
\end{aligned}
$$

For line A-C (Britton to Clark), we compute (from one observation session):

$$
\begin{aligned}
d\phi &= -0.000158510 \text{ deg} \\
d\lambda &= 0.158519430 \text{ deg} \\
dh &= 1.6263 \text{ m} \\
dH &= 1.910 \text{ m} \\
dN &= -0.2837 \text{ m}
\end{aligned}
$$

Since we have only three survey points, we must approximate the geoidal undulation difference surface with a plane. Using Lancaster's methods:

$$
\Delta = \det \begin{bmatrix} 1 & 0 & 0 \\ 1 & -.089487058 & .094818595 \\ 1 & -.000158510 & .158519430 \end{bmatrix} = -.014170408
$$

$$
\Delta^{-1} = -70.56959962
$$

and

$$
\frac{\partial N}{\partial \phi} = \Delta^{-1} \det \begin{bmatrix} 1 & 0 & 0 \\ 1 & -.2091 & .094818595 \\ 1 & -.2837 & .158519430 \end{bmatrix} = .440804353
$$

and

$$
\frac{\partial N}{\partial \lambda} = \Delta^{-1} \det \begin{bmatrix} 1 & 0 & 0 \\ 1 & -.089487058 & -.2091 \\ 1 & -.000158510 & -.2837 \end{bmatrix} = -1.789245193
$$

computing directly, we obtain:

$$
\begin{aligned}
\xi &= -0.82 \text{ seconds} \\
\eta &= 4.33 \text{ seconds} \\
\text{total magnitude} &= 4.41 \text{ seconds} \\
\text{azimuth} &= 100^\circ\ 43'\ 24.''5
\end{aligned}
$$

Rapp (1987, personal communication), has obtained the following results using a 360 degree model of the geoid (minimum wavelength of 50 km):

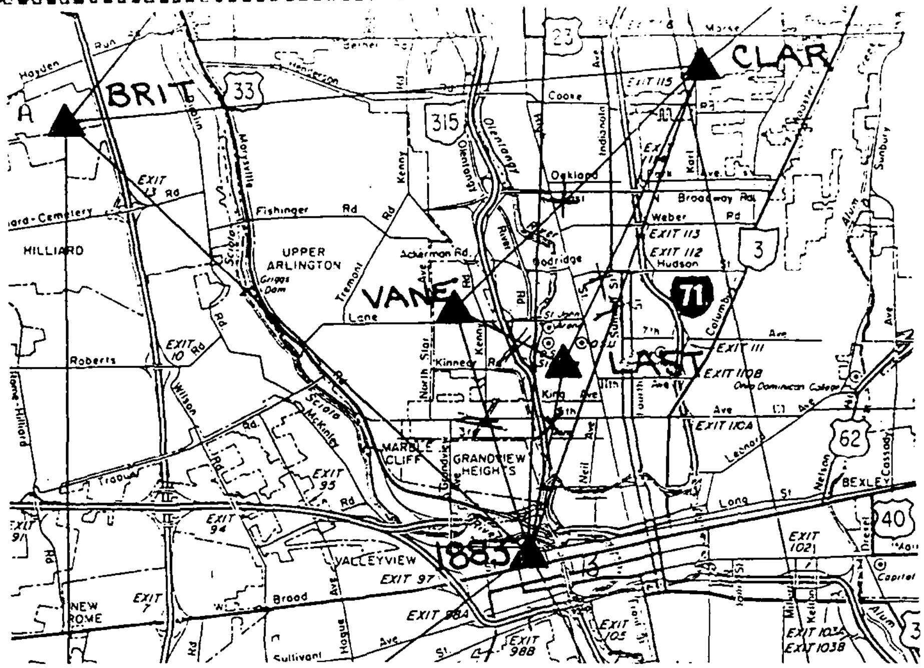
Station	ξ seconds	η seconds
A	1.36	2.57
B	1.26	1.90
C	2.03	2.18
average	1.55	2.22

Interpolation off a map of the geoid in Central Ohio (Engelis, Rapp) (See Figure 5), yeilds:

$$\xi = -0.5 \text{ seconds}$$
$$\eta = 4.0 \text{ seconds}$$

Although there is little correlation between GPS derived values and those obtained with Rapp's gravity model, (due mostly to the inability of Rapp's model to resolve deflections from wavelengths of less than 50 km in length), there is an extremely high correlation with the values obtained by interpolation from the geoidal map. The deflections computed with this GPS technique therefore appear to be valid and reliable (at least for this particular test area.)

The next step is to subject our GPS method to various errors (both absolute and relative) and see how they propagate into our solution.



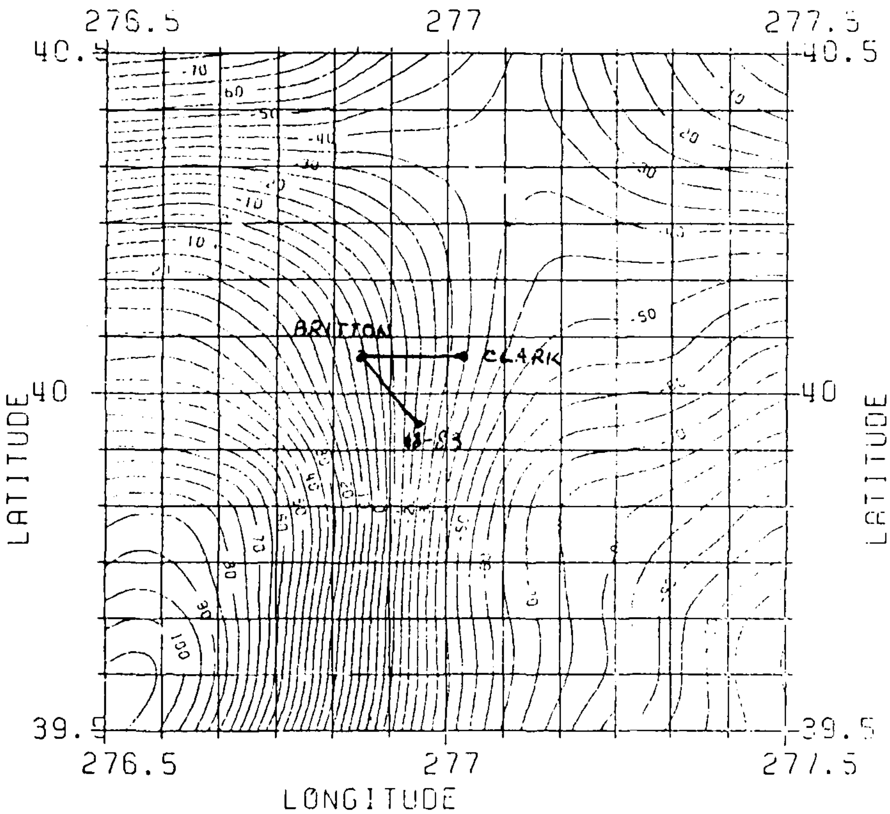


Figure 5.

Planimetric and geoidal maps of the survey test area in Central Ohio. The Britton - Clark - 18-83 "triangle" is indicated on both maps. The planimetric map is by the Ohio Department of Transportation (1985) and the map of the geoid is by Engelis and Rapp (1984).

Error #1:

Impose an absolute positioning error on Station A (Britton) of +20 meters in the X direction. (Stations B and C will of course experience the same shift.):

$$\frac{\partial N}{\partial \phi} = .415969848 \text{ m}$$

$$\frac{\partial N}{\partial \lambda} = -1.521793910 \text{ m}$$

$$\xi = -0.77 \text{ sec}$$

$$\eta = 3.68 \text{ sec}$$

Error #2:

Impose a relative positioning error on stations B and C of $(1\text{x}10^{-6})$ x (length of line):

Errors on line A-B:	x:	.009 m
	y:	-.005 m
	z:	.008 m
Errors on line A-C:	x:	.013 m
	y:	-.002 m
	z:	.000 m

$$\frac{\partial N}{\partial \phi} = .495630234 \text{ m}$$

$$\frac{\partial N}{\partial \lambda} = -1.724844248 \text{ m}$$

$$\xi = -0.92 \text{ sec}$$

$$\eta = 4.17 \text{ sec}$$

Error #3:

Impose an error on the absolute elevation above the geoid of point A. First order standards require that elevations be measured to a tolerance of 3mm x $(\text{distance})^{1/2}$ where the distance is in kilometers. (FGCC, 1980) The maximum error we may experience is then:

$$\text{line A–B} \quad \Delta H = .011 \text{ m}$$

$$\text{line A–C} \quad \Delta H = .011 \text{ m}$$

So we get:

$$\frac{\partial N}{\partial \phi} = .246832384 \text{ m}$$

$$\frac{\partial N}{\partial \lambda} = -1.856307937 \text{ m}$$

$$\xi = -0.46 \text{ sec}$$

$$\eta = 4.49 \text{ sec}$$

In tabular form: (all quantities are in seconds)

	'No' Errors	Abs. Error	Rel. Error	Lev. Error
ξ	-0.82	-0.77	-0.92	-0.46
η	4.33	3.68	4.17	4.49

We see that in almost every case, the errors we imposed did not deviate significantly from the original values. If our tolerance is ±.5 arcseconds then all but the absolute error in the meridian direction are thoroughly acceptable. Overall, the results are characterized by a striking consistency.

CONCLUSIONS

The similarity between the results obtained with the GPS algorithm described in this paper and those from Dr. Rapp's map of the geoid imply that the use of GPS for determining deflections of the vertical is a valid procedure. Of course, more data and more comparisons are required for even stronger confirmation. There is no theoretical reason why the GPS methods could not work exceptionally well in benign geoidal terrains. In regions of high variability, however, one must be very careful with one's conclusions.

The test bed that I used in my analysis was locally rather benign but regionally quite variable. Since the isarythms of the geoidal surface were only slightly curved in the test area, we can be reasonably sure of the quality of the results. Had the variability been more extreme, either more points or shorter lines would have been required. It seems as though in the future one will need at least some idea of the characteristics of the geoidal surface prior to planning a survey layout. With modern satellite methods and worldwide gravity models, however, this should not be a major stumbling block.

Overall, I would use GPS methods to determine deflections of the vertical in areas which: 1) have benign geoidal terrains; 2) commonly experience poor weather; 3) cannot be occupied for more than several hours (including hostile and unsafe regions); 4)

require immediate data; and 5) must be surveyed as cheaply as possible. I would not use GPS methods in areas which: 1) have extremely variable geoidal undulations; and/or 2) experience a high amount of radio frequency interference (RFI). Overall, the positive aspects of GPS surveying seem to outweigh the negative ones. Determinations of deflections of the vertical using GPS will probably become the method of choice for many agencies in the near future.

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The following pages are output from Dr. Goad's multireceiver Trimvec program which adjusts GPS data. Three sessions are included:

1.	Britton - Clark		5 March 1987
2.	Britton - 18-83	Session A	6 March 1987
3.	Britton - 18-83	Session B	6 March 1987

Default atmospheric values and adjusted antenna heights were used in these programs. All data was collected by the Franklin County Engineers Office. (Point of contact is Mr. Dean Ringle.)

10/26/87 10:27:33.78

TRIMBLE NAVIGATION, LTD
585 NORTH MARY AVENUE
SUNNYVALE, CALIFORNIA 94086
U.S.A.

BRITTON - CLARK

PROGRAM TRIMVEC
GPS RELATIVE POSITIONING SOLUTION
VERSION 87.106MB

File name: a:britclar.a64
Coordinate system - WGS-84

Type solution: Double difference

Start date/time: 1987/ 3/ 5 6:58:30. day of year 64 tow 370710.
Stop date/time: 1987/ 3/ 5 8:26:15. day of year 64 tow 375975.

Data available:

station: 1
sat: 3 |...|
sat: 9 |...|
sat:11 |...|
sat: 6 |...|
sat:13 | ....|

station: 2
sat: 3 |...|
sat: 9 |...|
sat:11 |...|
sat: 6 |...|
sat:13 | . .. .|

Ephemeris file used: a:brita064.eph

SATELLITE	AODE(hr.)	HEALTH	WEEK NO.	TOW(sec)	URA(m)
3	2.84	0	373	370590.00	8.0
9	6.83	0	373	370590.00	8.0
11	3.98	0	373	370590.00	8.0
6	4.55	0	373	370620.00	4.0
13	8.53	0	373	372660.00	8.0

Broadcast satellite clock correction values

prn	af0	af1	af2	toc
3	-.3044493496D-05	-.5684341886D-12	.0000000000D+00	.3744D+06
9	.3178184852D-04	.1659827831D-10	-.2775557562D-16	.3744D+06
11	.6031896919D-04	-.5115907697D-11	.0000000000D+00	.3744D+06
6	-.3404337913D-03	-.1546140993D-10	.0000000000D+00	.3744D+06
13	.9765336290D-04	.2273736754D-11	.0000000000D+00	.3780D+06

Message file for station 1

....

Station ID: BRITA064
Date: MAR. 4,1987

```
Session: 064A
Location: BRITTON RD. HILLIARD
Observer's name: BRIG
Receiver serial #: 4871
Antenna serial #: 0117
External computer serial #: 864C
Antenna height: 1.3770 meters (54.213 inches) - uncorrected to the vertical
BRITTON RD. HILLIARD
STA. BRIT
SESSION A

 ..........................................................................

 Origin of station 1 coordinates : Best C/A code tracking solution

   STATION (mark)  1

 input data file  1 : a:brita064.dat
  antenna height(m)      1.343

 met values used:  pressure(mb)          1010.0
                   temperature(deg C)      20.0
                   relative humidity(%)    50.0

   x (m)       584454.502      lat  (dms)  N    40     3   10.19823
   y (m)     -4854000.441      elon (dms)  E   276    51   56.69108
   z (m)      4082582.953      wlon (dms)  W    83     8    3.30892
                               ht (m)             163.5942

 Message file for station 2
 ..........................................................................
Station ID: CLARA064
Date: 5 MARCH 1987
Session: 064A
Location: DESALLES HIGH SCHOOL KARL ROAD FRANKLIN COUNTY STATION CLARK
Observer's name: KLS
Receiver serial #: 4874
Antenna serial #: 0116
External computer serial #: 5501905C
Antenna height: 1.5926 meters (62.701 inches) - uncorrected to the vertical
CLOUDY AND COLD. THE STATION OCCUPIED IS STATION CLARK WHICH WILL
BE USED AS PHOTO CONTROL.

 ..........................................................................

   STATION (mark)  2

 input data file  1 : a:clara064.dat
  antenna height(m)      1.563

 met values used:  pressure(mb)          1010.0
                   temperature(deg C)      20.0
                   relative humidity(%)    50.0

   x (m)       597883.283      lat  (dms)  N    40     3    9.62758
   y (m)     -4852377.340      elon (dms)  E   277     1   27.36113
   z (m)      4082570.527      wlon (dms)  W    82    58   32.63887
                               ht (m)             165.2205
```

Vector 1 originates at station 1 ends at station 2

Vector covariance matrix (cm**2) :

	dx(01)	dy(01)	dz(01)
dx(01)	.0030774		
dy(01)	.0032010	.0203249	
dz(01)	-.0022806	-.0124487	.0090608

Vector correlation matrix :

	dx(01)	dy(01)	dz(01)
dx(01)	1.0000000		
dy(01)	.4047446	1.0000000	
dz(01)	-.4318892	-.9173319	1.0000000

STATION 1 TO STATION 2

Rdop is .107680
slope distance (m) 13526.5208 sigma (cm) .064

normal section azimuth (dms) 90 1 24.78
vertical angle (dms) 0 -3 13.61

east(m) north(m) up(m) 13526.514 -5.560 -12.697

Delta lat(dms) 0 0 -.57064
Delta lon(dms) 0 9 30.67005
Delta ht(m) 1.6263

correlations for baseline 1:

	dx	dy	dz	trop	bias 1	bias 2	bias 3	bias 4
dx	1.0000000							
dy	.4047446	1.0000000						
dz	-.4318892	-.9173319	1.0000000					
trop	.0000000	.0000000	.0000000	1.0000000				
bias 1	.0000000	.0000000	.0000000	.0000000	1.0000000			
bias 2	.0000000	.0000000	.0000000	.0000000	.0000000	1.0000000		
bias 3	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	1.0000000	
bias 4	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	1.0000000

	Solution	Sigma
dx (m)	13428.781	.001
dy (m)	1623.101	.001
dz (m)	-12.426	.001
trop (%)	.000	.000
bias 1 (cycle)	.000	.000
bias 2 (cycle)	.000	.000
bias 3 (cycle)	.000	.000
bias 4 (cycle)	-14.000	.000

Results of integer bias search:
.81201 15.55343 28.66430

```
prn  9                 0                  0                  -1

prn 11                 0                  0                  -1

prn  6                 0                  0                  -1

prn 13                 0                 -1                  -1

Ratio sum-of-squares(2) to sum-of squares(1)      19.15



 Interval between epochs (sec)    15
 Epoch increment      1
 Number of measurements used in solution    918
 Number of measurements rejected            145
 RMS (cycles)        .033

 Elevation mask (deg)      15.0
 Edit multiplier    3.5
 Modified Hopfield troposphere model used

Best tracking C/A code positions
Station  1
Pdop        15.1
  x (m)       584454.625      lat  (dms)  N    40     3  10.19823
  y (m)     -4854001.462      elon (dms)  E   276    51  56.69108
  z (m)      4082583.817      wlon (dms)  W    83     8   3.30892
                              ht  (m)          164.9372
clock offset(s)             .67806059D-03
freq  offset(s/s)           .16051102D-10

 Code calibration(m)     Carrier calibration(m)
1 - 2         -.5271                     -.0009
1 - 3         -.1090                      .0005
1 - 4         -.3663                     -.0006
1 - 5        -1.0829                     -.0016
Station  2
Pdop        12.7
  x (m)       597904.922      lat  (dms)  N    40     3   9.51469
  y (m)     -4852390.070      elon (dms)  E   277     1  28.20151
  z (m)      4082578.823      wlon (dms)  W    82    58  31.79849
                              ht  (m)          182.2562
clock offset(s)           -.77472944D-01
freq  offset(s/s)           .60433110D-08

 Code calibration(m)     Carrier calibration(m)
1 - 2         -.1251                      .0007
1 - 3         2.2444                      .0005
1 - 4         -.5557                      .0005
1 - 5         2.1747                      .0007
```

10/26/87 10:48:27.18

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U.S.A.

BRITTON - 18-83
SESSION A

PROGRAM TRIMVEC
GPS RELATIVE POSITIONING SOLUTION
VERSION 87.106MB

File'name: a:brit1883.a65
Coordinate system - WGS-84

Type solution: Double difference

Start date/time: 1987/ 3/ 6 6:52:30. day of year 65 tow 456750.
Stop date/time: 1987/ 3/ 6 8:19:45. day of year 65 tow 461985.

Data available:

station: 1
sat: 3 ¦...¦
sat: 6 ¦...¦
sat: 9 ¦...¦
sat:11 ¦...¦
sat:13 ¦ ....¦

station: 2
sat: 3 ¦...¦
sat: 6 ¦...¦
sat: 9 ¦...¦
sat:11 ¦...¦
sat:13 ¦ ....¦

Ephemeris file used: a:brita065.eph

SATELLITE	AODE(hr.)	HEALTH	WEEK NO.	TOW(sec)	URA(m)
3	2.84	0	373	456630.00	5.7
6	5.69	0	373	456660.00	4.0
9	6.83	0	373	456630.00	8.0
11	10.24	0	373	456630.00	8.0
13	8.53	0	373	458820.00	5.7

Broadcast satellite clock correction values

prn	af0	af1	af2	toc
3	-.3090128303D-05	-.5684341886D-12	.0000000000D+00	.4608D+06
6	-.3417823464D-03	-.1591615728D-10	.0000000000D+00	.4608D+06
9	.3316905349D-04	.1602984412D-10	-.2775557562D-16	.4608D+06
11	.5984678864D-04	-.5456968211D-11	-.2775557562D-16	.4608D+06
13	.9784381837D-04	.2160049917D-11	.0000000000D+00	.4644D+06

Message file for station 1
....
Station ID: BRITA065
Date: MAR.6,1987

```
Session:
Location: BRITTON RD. HILLIARD
Observer's name: BRIG
Receiver serial #: 4871
Antenna serial #: 0117
External computer serial #: 864C
Antenna height: 1.3560 meters (53.386 inches) - uncorrected to the vertical
BRITTON RD. HILLIARD
STA. BRIT
SESSION A

 ...............................................................................

 Origin of station 1 coordinates : Best C/A code tracking solution

   STATION (mark)  1

 input data file  1 : a:brita065.dat
  antenna height(m)      1.321

 met values used:  pressure(mb)          1010.0
                   temperature(deg C)      20.0
                   relative humidity(%)    50.0

   x (m)       584396.324      lat  (dms)  N    40     3  10.64840
   y (m)     -4854051.276      elon (dms)  E   276    51  53.99783
   z (m)      4082637.675      wlon (dms)  W    83     8   6.00217
                               ht  (m)           232.1165

 Message file for station 2
 ...............................................................................
Station ID: 1883a065
Date: 03-06-87
Session: 065
Location: mon 1883 in park at corner of central & mckinely
Observer's name: dlp
Receiver serial #: 4872
Antenna serial #: 2704a00128
External computer serial #: 5501536c
Antenna height: 1.6080 meters (63.307 inches) - uncorrected to the vertical
warm 40 degrees roughly & very calm

 ...............................................................................

   STATION (mark)  2

 input data file  1 : a:1883a065.dat
  antenna height(m)      1.579

 met values used:  pressure(mb)          1010.0
                   temperature(deg C)      20.0
                   relative humidity(%)    50.0

   x (m)       593197.534      lat  (dms)  N    39    57  48.49478
   y (m)     -4859377.496      elon (dms)  E   276    57  35.34498
   z (m)      4074992.037      wlon (dms)  W    83     2  24.65502
                               ht  (m)           178.0629
```

Vector 1 originates at station 1 ends at station 2

Vector covariance matrix (cm**2) :

	dx(01)	dy(01)	dz(01)
dx(01)	.0019437		
dy(01)	.0024505	.0075512	
dz(01)	-.0020715	-.0047747	.0042756

Vector correlation matrix :

	dx(01)	dy(01)	dz(01)
dx(01)	1.0000000		
dy(01)	.6396375	1.0000000	
dz(01)	-.7185848	-.8403219	1.0000000

STATION 1 TO STATION 2

Rdop is .095704
slope distance (m) 12817.3986 sigma (cm) .041

normal section azimuth (dms) 140 47 47.44
vertical angle (dms) 0-17 57.31

east(m) north(m) up(m) 8101.466 -9932.144 -66.944

Delta lat(dms) 0 -5 22.15362
Delta lon(dms) 0 5 41.34716
Delta ht(m) -54.0536

correlations for baseline 1:

	dx	dy	dz	trop	bias 1	bias 2	bias 3	bias 4
dx	1.0000000							
dy	.6396375	1.0000000						
dz	-.7185848	-.8403219	1.0000000					
trop	.0000000	.0000000	.0000000	1.0000000				
bias 1	.0000000	.0000000	.0000000	.0000000	1.0000000			
bias 2	.0000000	.0000000	.0000000	.0000000	.0000000	1.0000000		
bias 3	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	1.0000000	
bias 4	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	1.0000000

	Solution	Sigma
dx (m)	8801.211	.000
dy (m)	-5326.220	.001
dz (m)	-7645.638	.001
trop (%)	.000	.000
bias 1 (cycle)	.000	.000
bias 2 (cycle)	.000	.000
bias 3 (cycle)	.000	.000
bias 4 (cycle)	.000	.000

Results of integer bias search:

	4.07169	16.41816	25.36119
prn 6	0	-1	0

prn 9	0	0	-1
prn 11	0	0	0
prn 13	0	0	0

Ratio sum-of-squares(2) to sum-of-squares(1) 4.03

Interval between epochs (sec) 15
Epoch increment 1
Number of measurements used in solution 1012
Number of measurements rejected 172
RMS (cycles) .025

Elevation mask (deg) 15.0
Edit multiplier 3.5
Modified Hopfield troposphere model used

Best tracking C/A code positions
Station 1
Pdop 14.3

x (m)	584396.445	lat (dms)	N	40	3	10.64840
y (m)	-4854052.280	elon (dms)	E	276	51	53.99783
z (m)	4082638.525	wlon (dms)	W	83	8	6.00217
		ht (m)		233.4375		

clock offset(s) -.69907204D-01
freq offset(s/s) -.26396778D-08

Code calibration(m)		Carrier calibration(m)
1 - 2	-.5486	-.0013
1 - 3	.1161	.0000
1 - 4	-.2448	-.0008
1 - 5	-.8381	-.0022

Station 2
Pdop 13.1

x (m)	593259.704	lat (dms)	N	39	57	48.13617
y (m)	-4859339.368	elon (dms)	E	276	57	38.13979
z (m)	4074952.203	wlon (dms)	W	83	2	21.86021
		ht (m)		129.2440		

clock offset(s) .14451944D-03
freq offset(s/s) .30199690D-09

Code calibration(m)		Carrier calibration(m)
1 - 2	.1555	.0003
1 - 3	1.5778	-.0014
1 - 4	-1.0203	-.0001
1 - 5	-.9131	-.0016

10/27/87   12:40:18.97

TRIMBLE NAVIGATION, LTD
585 NORTH MARY AVENUE
SUNNYVALE, CALIFORNIA 94086
U.S.A.

BRITTON - 18-B

PROGRAM TRIMVEC
GPS RELATIVE POSITIONING SOLUTION
VERSION 87.106MB

SESSION 3

File'name: aristov.fix
Coordinate system - WGS-84

Type solution: Double difference

Start date/time: 1987/ 3/ 6   8:47:45.   day of year 65   tow   463665.
Stop date/time: 1987/ 3/ 6   10:16:60.   day of year 65   tow   469020.

Data available:

station: 1
sat: 9 |.................................... |
sat:11 |..|
sat:12 |..|
sat:13 |..|
sat: 3 |..|

station: 2
sat: 9 |.................................... |
sat:11 |..|
sat:12 |..|
sat:13 |..|
sat: 3 |..|

Ephemeris file used: a:britb065.eph

SATELLITE	AODE(hr.)	HEALTH	WEEK NO.	TOW(sec)	URA(m)
9	8.53	0	373	463530.00	8.0
11	11.95	0	373	463500.00	5.7
12	6.26	0	373	463530.00	5.7
13	3.41	0	373	463530.00	5.7
3	4.55	0	373	463560.00	4.0

Broadcast satellite clock correction values

prn	af0	af1	af2	toc
9	.3328500316D-04	.1602984412D-10	-.2775557562D-16	.4680D+06
11	.5980860442D-04	-.5456968211D-11	-.2775557562D-16	.4680D+06
12	.4641688429D-03	.5570655048D-11	.0000000000D+00	.4680D+06
13	.9786570445D-04	.2387423592D-11	.0000000000D+00	.4680D+06
3	-.3093853593D-05	-.5684341886D-12	.0000000000D+00	.4680D+06

Message file for station 1

....

Station ID: BRITB065
Date: MAR.6,1987

Session: 065B
Location: BRITTON RD. HILLIARD
Observer's name: BRIG
Receiver serial #: 4871
Antenna serial #: 0117
External computer serial #: 864C
Antenna height: 1.3122 meters (51.661 inches) - uncorrected to the vertical
BRITTON RD. HILLIARD
STA. BRIT
SESSION B

....

Origin of station 1 coordinates : Best C/A code tracking solution

STATION (mark) 1

input data file 1 : a:britb065.dat
antenna height(m) 1.276

met values used:	pressure(mb)	1010.0
	temperature(deg C)	20.0
	relative humidity(%)	50.0

x (m)	584399.071	lat (dms)	N	40	3	10.65202
y (m)	-4854054.663	elon (dms)	E	276	51	54.09583
z (m)	4082640.924	wlon (dms)	W	83	8	5.90417
		ht (m)		237.0324		

Message file for station 2

....

Station ID: 1883b065
Date: 03-06-87
Session: b065
Location: mon 1883 at corner of central &mckinely
Observer's name: dlp
Receiver serial #: 4872
Antenna serial #: 2704a00128
External computer serial #: 5501536c
Antenna height: 1.5420 meters (60.709 inches) - uncorrected to the vertical

....

STATION (mark) 2

input data file 1 : a:1883b065.dat
antenna height(m) 1.512

met values used:	pressure(mb)	1010.0
	temperature(deg C)	20.0
	relative humidity(%)	50.0

x (m)	593200.285	lat (dms)	N	39	57	48.49871
y (m)	-4859380.869	elon (dms)	E	276	57	35.44279
z (m)	4074995.281	wlon (dms)	W	83	2	24.55721
		ht (m)		182.9681		

Vector 1 originates at station 1 ends at station 2

Vector covariance matrix (cm**2) :

	dx(01)	dy(01)	dz(01)
dx(01)	.0003795		
dy(01)	-.0008262	.0074935	
dz(01)	.0000941	-.0021332	.0014925

Vector correlation matrix :

	dx(01)	dy(01)	dz(01)
dx(01)	1.0000000		
dy(01)	-.4898924	1.0000000	
dz(01)	.1250329	-.6378768	1.0000000

STATION 1 TO STATION 2

Rdop is .080179
slope distance (m) 12817.3983 sigma (cm) .037

normal section azimuth (dms) 140 47 47.40
vertical angle (dms) 0-17 57.48

east(m) north(m) up(m) 8101.467 -9932.142 -66.955

Delta lat(dms) 0 -5 22.15332
Delta lon(dms) 0 5 41.34697
Delta ht(m) -54.0642

correlations for baseline 1:

	dx	dy	dz	trop	bias 1	bias 2
	bias 3	bias 4				
dx	1.0000000					
dy	-.4898924	1.0000000				
dz	.1250329	-.6378768	1.0000000			
trop	.0000000	.0000000	.0000000	1.0000000		
bias 1	.0000000	.0000000	.0000000	.0000000	1.0000000	
bias 2	.0000000	.0000000	.0000000	.0000000	.0000000	1.0000000
bias 3	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
	1.0000000					
bias 4	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
	.0000000	1.0000000				

	Solution	Sigma
dx (m)	8801.214	.000
dy (m)	-5326.207	.001
dz (m)	-7645.643	.000
trop (%)	.000	.000
bias 1 (cycle)	.000	.000
bias 2 (cycle)	.000	.000
bias 3 (cycle)	.000	.000
bias 4 (cycle)	.000	.000

Results of integer bias search:
1.76043 8.66400 10.90994

prn 11 0 1 0

prn 12	0	1	1
prn 13	0	1	0
prn 3	0	1	0

Ratio sum-of-squares(2) to sum-of-squares(1) 4.92

Interval between epochs (sec) 15
Epoch increment 1
Number of measurements used in solution 1060
Number of measurements rejected 129
RMS (cycles) .024

Elevation mask (deg) 15.0
Edit multiplier 3.5
Modified Hopfield troposphere model used

Best tracking C/A code positions
Station 1
Pdop 6.1

x (m)	584399.188	lat (dms)	N	40	3	10.65202
y (m)	-4854055.632	elon (dms)	E	276	51	54.09583
z (m)	4082641.745	wlon (dms)	W	83	8	5.90417
		ht (m)		238.3084		

clock offset(s) -.68906956D-01
freq offset(s/s) -.56754055D-09

	Code calibration(m)	Carrier calibration(m)
1 - 2	-.4253	-.0019
1 - 3	.1072	-.0005
1 - 4	-.0536	-.0012
1 - 5	-.6754	-.0033

Station 2
Pdop 6.1

x (m)	593201.878	lat (dms)	N	39	57	48.32256
y (m)	-4859412.488	elon (dms)	E	276	57	35.34799
z (m)	4075014.655	wlon (dms)	W	83	2	24.65201
		ht (m)		219.6162		

clock offset(s) -.69159230D-01
freq offset(s/s) -.19168765D-09

	Code calibration(m)	Carrier calibration(m)
1 - 2	.1447	.0003
1 - 3	-.2377	-.0012
1 - 4	-.1537	-.0000
1 - 5	-.0929	-.0013